

Cobb-Douglas Agri-Meteorological Model Applied to the Reference Evapotranspiration for Benin Subbasin of Niger River

Volynski R., Ben-Asher J, Selezov I. , Gulko N

Abstract This paper is concerned with using of Cobb-Douglas agri-meteorological model for the environmental data obtained for the rice fields placed in the Benin Subbasin for Niger River for seventy years of observation. Mathematical model was developed using Matlab software for the multiple non-linear regression with the coefficient of determination $Rsq=0.9876$, i.e. with high goodness of fitting. It is concluded that for the extremely complicated environmental conditions for considering rice fields reasonable use of water resources needs in the individual regional formula for evapotranspiration similar to that developed in Poland recently. Presented Cobb-Douglas model could be a step to obtain this aim

Index Terms—Cobb-Douglas model, ET₀, production function

1 INTRODUCTION

1.1 Introduction

First functional relationship between the amount of the spent resources and the amount of obtained production has been examined in the paper "Production Theory" duplicated in the year 1928 by two American scientists: economist Peter Douglas and mathematician George Cobb. The paper was an attempt to define empirically the effect of the quantities of the capital (K) and Labor (L) spent for production (Q). The statistical data representing the USA manufacturing for the years 1899-1922 were taken under consideration. The following aims of the work were posed:

1. Define the functions accurately expressing quantitative relationships between three selected production characteristics.
2. Find partition coefficients for the all functions.
3. Check the accuracy of the solution by means of its comparison with measured data.

In this work a good approximation of the measured data by the fitted data was obtained.

The Cobb-Douglas type of production function has been used most frequently in order to express mathematically the input-output relationship of the economical variables. The model allows to analyze the input-output dependences for different areas in the form

$$Y = aX_1^{b_1}X_2^{b_2}X_3^{b_3}\dots X_n^{b_n} \quad (1)$$

Where the coefficient (b_1, b_2, \dots, b_n) associated with each factor input (X_1, X_2, \dots, X_n) corresponds to what

is known in economic terminology as the elasticity of production of that factor. The equation (1) is transformed into logarithms and reduces to the simple linear equation

$$\log Y = \log a + b_1 \log X_1 + \dots + b_n \log X_n \quad (2)$$

which can be solved by the familiar means of least squares. This property simplifies the computational work involved in the use of the function. The function has a number of other properties, which, since they correspond to well-known economic concepts and axioms, enhance its value in economic context.

The model was used to analyze the linkages between input and output in such areas as economics, finances, ecology [7]. Currently Cobb-Douglas method is adopted by the agricultural researchers [1],[2],[3],[8].

In agronomy research has provided scientific information for developing efficient irrigation practices to improve crop yield and water use effectivity.

In present work we intend to use the Cobb-Douglas production function to deal with the Agrometeorological model for the Reference Evapotranspiration ET₀, which is a very important variable for the agricultural needs. The work on transpiration are carried constantly all over the world and computational methods are continuously being improved. In particular, the aim of this work consists in obtaining regional formula describing transpiration in the regions of the country. For example, recently

has been published the formula obtained by Polish scientists for one of the regions of Poland [9].

2. Aim of the work

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The aim of the present work is in the expanding of the Cobb-Douglas approach on the combined Agricultural-Meteorological model

3. Materials and methods

As a sample case the available set of the data from the rice field placed at the Benin is under consideration [4].

3.1 Site description

The rice field in Benin Sub Basin of Niger River is located within a wide West African Niger River basin (Figure 1) characterized by a tropical continental climate with two seasons for a year . It covers an area of 46 000 km², about 40% of Benin total area (112 622 km²). The meteorological input data may be summarized as follows. The mean monthly temperature varies between 32.5°C in April to 25.2°C in December and the average annual rainfall is 849 mm. A five-

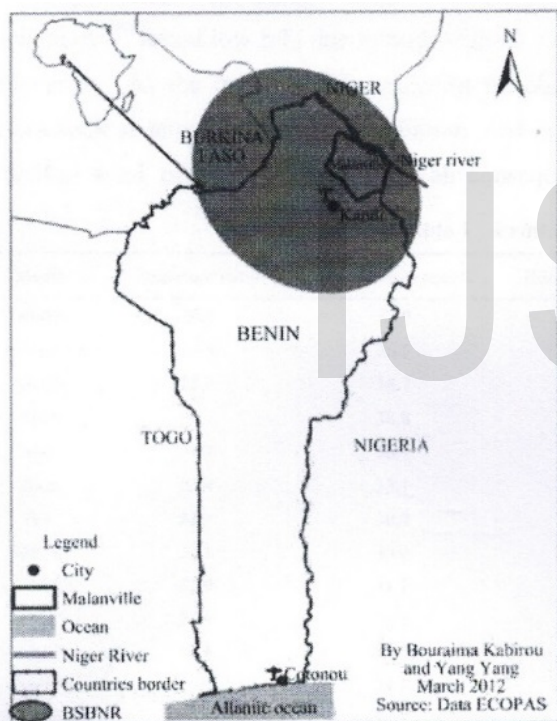


Fig.1. Location of Benin Sub Basin of Niger River (BSBNR).

month rain y season is from April to October by a dry season of seven-

Two distinct and powerful winds: Alize maritime (monsoon) and Alize continental (harmattan) vary over time with different speeds: the monsoon wind blows from April to November in the SW direction with speed ranging from 3 m/s in April to 2 m/s in mid-October. The harmattan blows from November to March in the N or NE direction [4].

3.2 Input data matrix

Agrometeorological Cobb-Douglas model for the ET₀ using a multiple non-linear Regression starts with the TABLE 1, which contains average results of the observation for meteorological variables obtained from the year 1942 to 2012 [4].

month (November-March). The months from June to September are the wettest with 90% cumulative annual rainfall, or an average 731 mm. In fact, in August alone accounts for about 30%, or an average of 257 mm of the total annual rainfall. The annual humidity varies from 25% to 80% and 25% to 80% and average 60%. Daily sunshine length varies between 5 hours to 11 hours.

TABLE 1.
INPUT TABLE

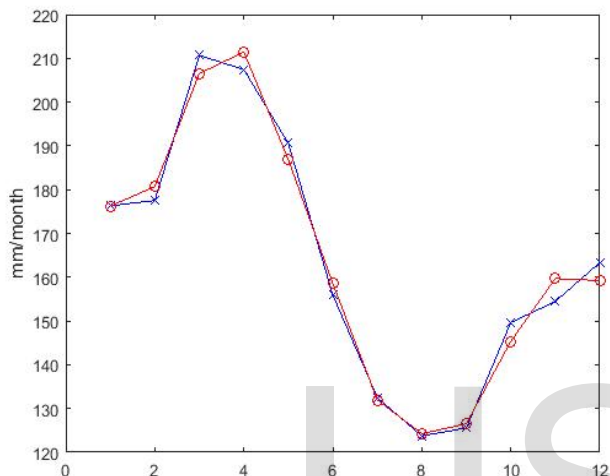
FOR COBB-DOUGLAS MODEL

N	1	2	3	4	5	6	7	8
G1=	16.6	33.9	31	179	8.6	19.6	176.30	
2	19.8	36.5	28	179	8.5	20.9	177.53	
3	24.0	38.7	35	186	7.6	20.8	210.69	
4	26.2	38.8	49	200	8.0	21.9	207.43	
5	25.0	36.1	62	196	8.3	22.0	190.61	
6	23.4	33.1	72	176	8.1	21.3	155.92	
7	22.4	30.9	78	120	6.7	19.3	132.52	

8	22.1	30.0	80	107	6.1	18.7	123.73
9	21.9	31.1	78	103	6.9	19.7	125.56
10	22.1	34.1	68	117	8.5	21.1	149.63
11	18.6	35.5	49	130	9.2	20.7	154.42
12	16.4	34.2	37	155	9.0	19.6	163.35];

Input Table is a matrix containing 12 lines and 8 rows, which are of the following meanings:

- 1-Month from Januar to December;
- 2-Temperature min,°C;
- 3-Temperature max,°C;



- 4- Humidity %;
- 5- Wind,km/ day;
- 6-Sunshine, hours;
- 7- Radiation,MJ/ m²/ day;
- 8- ET0,mm/ month.

Reference transpiration ET0 for the 8th row for the Table 1 was calculated using FAO Penman-Monteith method described in the decision support software CROPWAT 8.0 [5],[6].

4 Solution

4.1 X-function

Using data from the input Table 1 the following variables are introduced:

M=G1(:,1)-MONTH
 TMIN=G1(:,2)-MIN. TEMPERATURE
 TMAX=G1(:,3)-MAX. TEMPERATURE
 HUM=G1(:,4)- HUMIDITY
 WIND=G1(:,5)-WIND SPEED
 HOURS=G1(:,6)-SUNSHINE HOURS
 RAD=G1(:,7)-RADIATION

Further was developed the function:

$X=[\text{ones}(N,1) \log(\text{TMIN}) \log(\text{TMAX}) \dots \log(\text{HUM}) \log(\text{WIND}) \log(\text{HOURS}) \log(\text{RAD})];$

For this function is applied Least Square algorithm and numerical solution is obtained using MATLAB program .

Fig.2 Measured and fitted ET0 data with RSQ=0.9876

5.Results

There are many different factors that affect ET0 as agricultural output. Among them: temperature, humidity, wind, sunshine, and radiation. If we can predict evaporation rates (ET0), so it will be able to estimate the water demands of the crop. This may help us to determine whether or not to irrigate. The prediction of temperature and wind conditions can give a clue to how strong the evaporation rates will be. In this work we built logarithmic X-function, based on restricted number of factors that affect ET0. Graph on Fig2 demonstrates measured and fitted ET0 data with coefficient of the determination RSQ=0.9876 .That means that there is a high goodness of fitting in the adopted Cobb-Douglas model.

6.Conclusions

This study is to analyze the temporal variation of the agricultural output -ET0 and meteorological input in the framework of the Cobb-Douglas approach with RSQ=0.9876.For the analysis we use as integrate data for 70 years accomulated in Benin Sub Basin of Niger River.It is concluded here that that creation of the individual regional formula for ET0, as it has been done in Poland , should be effective also in Benin Sub Basin of Niger River in order to improve the water use.

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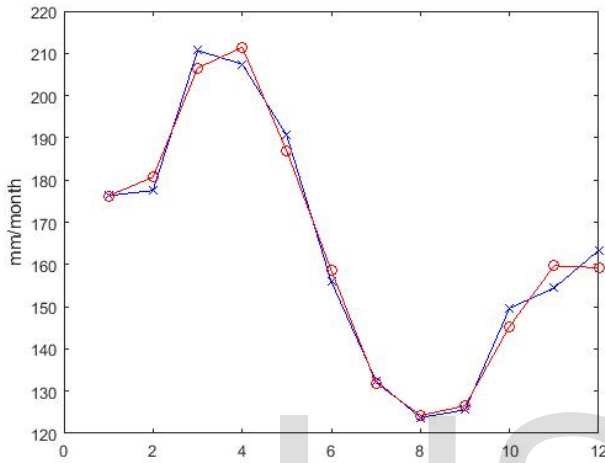
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